Week 2
MATH 34B
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10.3 An artery has a circular cross section of radius 2 millimeters. The speed at which blood flows along the artery fluctuates as the heart beats. The speed after t seconds is $20 + 6\sin(2\pi t)$ meters per second.

What volume of blood passes along the artery in one second?

$$V = \pi r^2 h = \pi (2)^2 \left[\int_0^1 20 + 6\sin(2\pi t)\,dt\right] \cdot 1000$$

b/c meters!

$$= \pi \cdot 4 \cdot \left[20t - \frac{6}{2\pi}\cos(2\pi t)\Big|_0^1\right] \cdot 1000$$

$$= \pi \cdot 4 \cdot 20 \cdot 1000.$$

10.5 (a) Use the product rule to find the derivative of $(3x+3)(\cancel{2x5})$. 2x-5

(b) Now multiply out and work out the derivative again and check that the answers agree.

(c) Now see what you get when you multiply the derivative of (3 x +3) with the derivative of (2 x-5). Note how different this is and understand why when taking the derivative of a product, you MUST use the CORRECT PRODUCT RULE!

a) $$\frac{d}{dx}(3x+3)(2x-5) = \left[\frac{d}{dx}(3x+3)\right](2x-5) + (3x+3)\frac{d}{dx}(2x-5)$$

$$= 3(2x-5) + 2(3x+3).$$

$$= \cancel{12x-9}.$$

12x - 9

b) $$\frac{d}{dx}(3x+3)(2x-5)$$

$$= \frac{d}{dx}(6x^2-9x-15) = 12x-9$$

c) $$\left[\frac{d}{dx}(3x+5)\right]\left[\frac{d}{dx}(2x-5)\right] = 3 \cdot 2 = 6$$

10.8 (a) $e^{3x}\ln(x)$

(b) $(9x^8-3)\sin(3x)$

(c) $\sin(2x)\cos(6x)$

(d) $(8x^7+2x^5)\sin(7x)$

(e) $4e^{7x}\sin(3x)$

a) $\cancel{\frac{d}{dx}((3x) \cdot \ln x)}$

$\frac{d}{dx}(e^{3x})\ln x + e^{3x}\frac{d}{dx}\ln x$

$= 3e^{3x} \ln x + e^{3x}\cdot\frac{1}{x}$

b) $\left[\frac{d}{dx}(9x^8-3)\right]\sin 3x + (9x^8-3)\frac{d}{dx}\sin 3x$

$= 72x^7\sin 3x + (9x^8-3)(\cos 3x)\cdot 3$

c) $\left[\frac{d}{dx}\sin 2x\right]\cos 6x + \sin 2x\left[\frac{d}{dx}\cos 6x\right]$

$= 2\cos 2x \cdot \cos 6x + \sin 2x\left[-6\sin 6x\right]$

d) $\left[\frac{d}{dx}(8x^7+2x^5)\right]\sin 7x + (8x^7+2x^5)\frac{d}{dx}\sin 7x$

$= (56x^6+10x^4)\sin 7x + (8x^7+2x^5)\cdot 7\cos 7x$

e) $\left[\frac{d}{dx}4e^{7x}\right]\sin 3x + 4e^{7x}\frac{d}{dx}\sin 3x$

$= 28e^{x}\sin 3x + 4e^{7x}\cdot(3\cos 3x)$ //

9.5 Differentiate

(a) 10^x

(b) $5 \cdot 2^x$

a) $10^x = e^{\ln(10^x)} = e^{x \ln 10}$

$\Rightarrow \frac{d}{dx} 10^x = \frac{d}{dx} e^{x \ln 10} = (\ln 10) e^{x \ln 10} = (\ln 10) 10^x$

b) $2^x = e^{x \cdot \ln 2}$

$\frac{d}{dx} 5 \cdot 2^x = 5 \cdot \frac{d}{dx} 2^x = 5 \cdot \frac{d}{dx} e^{x \ln 2} = 5 \cdot \ln 2 \cdot e^{x \ln 2}$

$= 5 \cdot (\ln 2) \cdot 2^x$

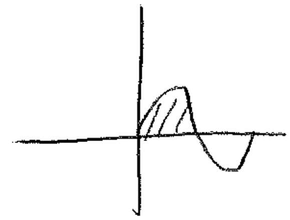
9.13 Integrate: $\int_0^{\pi/10} \sin(5x) dx$

$= -\frac{1}{5} \cos(5x) \Big|_0^{\pi/10}$

$= \underbrace{-\frac{1}{5} \cos\left(\frac{5\pi}{10}\right)}_{=0} - \underbrace{\left(-\frac{1}{5} \overbrace{\cos(0)}^{=1}\right)}_{=-1/5}$

$= 1/5$

9.14 Find the area under one arch of the graph $y = \sin(6x)$.



period of $\sin(6x)$ is $\frac{2\pi}{6} = \pi/3$

so one of humps is from 0 to $\pi/6$.

so...

$$\int_0^{\pi/6} \sin 6x = -\frac{1}{6} \cos 6x \Big|_0^{\pi/6}$$

$$= -\frac{1}{6} \cos \pi - \left(-\frac{1}{6} \cos 0\right)$$

$$= \frac{1}{6} + \frac{1}{6} = 1/3.$$

B.1 Find a point x that maximizes $e^{(\sin^2(x)+\cos^2(x))^3}$. How many of them are there?

$$e^{(\sin^2 x + \cos^2 x)^3} = e^{1^3} = e.$$

$\Rightarrow$ our function is constant!!!